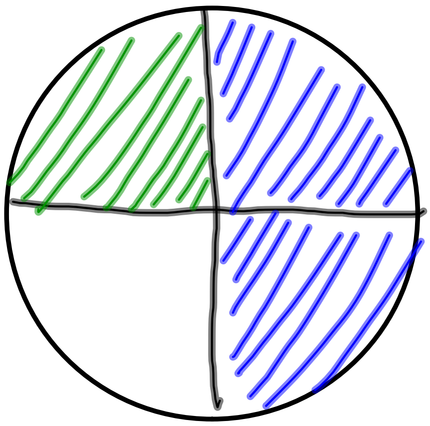
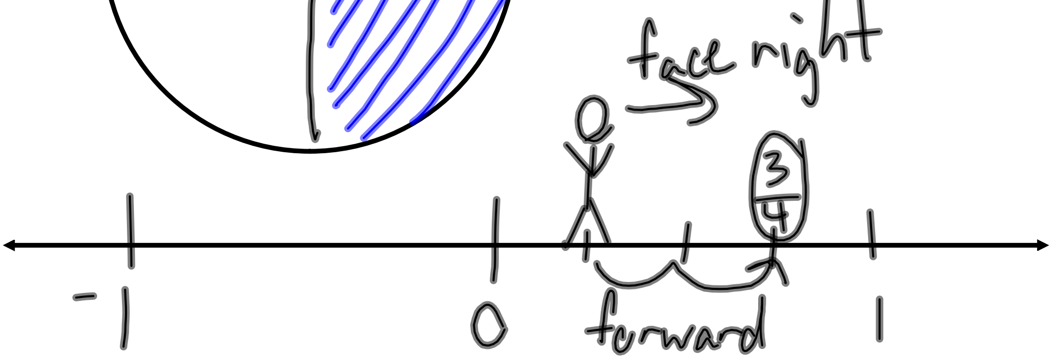
6.2: Addition and Subtraction of Rational Numbers

Definition: If $\frac{a}{b}$ and $\frac{c}{b}$ are rational numbers, then $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$.

Example: Draw a figure and a number line to represent $\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$.

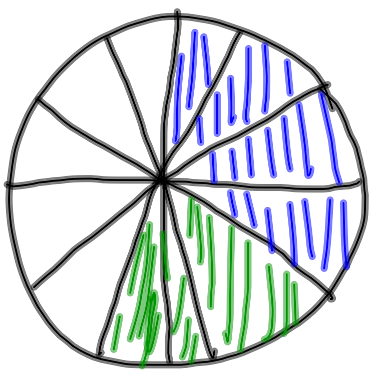




Example: Find a method of evaluating $\frac{1}{3}+\frac{1}{4}$.

$$\frac{1}{3}=\frac{1\cdot 4}{3\cdot 4}=\frac{4}{12} \qquad \frac{1}{4}=\frac{1\cdot 3}{4\cdot 3}=\frac{3}{12}$$

$$\frac{1}{3}+\frac{1}{4}=\frac{4}{12}+\frac{3}{12}=\frac{4+3}{12}=\frac{7}{12}$$



6.2 Completed Notes

Theorem: If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$.

(Note: In practice, we usually find the least common denominator instead of just using this formula.)

Proof: $\frac{a}{b}=\frac{a\cdot d}{b\cdot d}=\frac{ad}{bd}$, $\frac{c}{d}=\frac{c\cdot b}{d\cdot b}=\frac{bc}{bd}$

$$\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$$

Example: Calculate the following sums. Simplify your answers.

(a) $\frac{3}{10}+\frac{4}{15}=\frac{3\cdot 3}{10\cdot 3}+\frac{4\cdot 2}{15\cdot 2}=\frac{9}{30}+\frac{8}{30}=\boxed{\frac{17}{30}}$

LCM is 30

$$\frac{3}{10}+\frac{4}{15}=\frac{45+40}{150}=\frac{85}{150}=\frac{17}{30}$$

(b) $\frac{3}{9}+\frac{2}{6}=\frac{3\cdot 2}{9\cdot 2}+\frac{2\cdot 3}{6\cdot 3}=\frac{6}{18}+\frac{6}{18}=\frac{12}{18}=\boxed{\frac{2}{3}}$

LCM is 18

$$\frac{3}{9}+\frac{2}{6}=\frac{18+18}{54}=\frac{36}{54}=\frac{2}{3}$$

(reduce first) $\frac{1}{3}+\frac{1}{3}=\boxed{\frac{2}{3}}$

Definition: A <u>mixed number</u> is a number of the form $a\frac{b}{c}$, where a is an integer and $\frac{b}{c}$ is a proper fraction. The notation means $a\frac{b}{c} = a + \frac{b}{c}$.

A mixed number is a rational number, so we should be able to write it as $\frac{a}{b}$.

Example: Write the following numbers in the $\frac{a}{b}$ form.

(a) $2\frac{1}{4} = 2 + \frac{1}{4} = \frac{2}{1} + \frac{1}{4} = \frac{2\cdot 4}{1\cdot 4} + \frac{1}{4} = \frac{8}{4} + \frac{1}{4} = \boxed{\frac{9}{4}}$

$2\frac{+1}{\times 4} = \frac{9}{4}$

(b) $^{-}3\frac{2}{5} = -\left(3 + \frac{2}{5}\right) = -3 - \frac{2}{5} = \frac{-15}{5} - \frac{2}{5} = \frac{-17}{5}$

$-\left(3\frac{+2}{\times 5}\right) = -\frac{17}{5}$

Example: Change the following fractions to mixed numbers.

(a) $\frac{22}{7} = 3\frac{1}{7}$

wholes

$$\begin{array}{r} 3 \\ 7\overline{)22} \\ -21 \\ \hline 1 \end{array}$$

denominator

numerator

(b) $\frac{64}{19} = 3\frac{7}{19}$

$$\begin{array}{r} 3 \\ 19\overline{)64} \\ -57 \\ \hline 7 \end{array}$$

6.2 Completed Notes

Example: Calculate the following sums. Leave your answers as mixed numbers.

(a) $2\frac{3}{7} + 1\frac{11}{14} = \left(2+\frac{3}{7}\right)+\left(1+\frac{11}{14}\right)$

$= (2+1) + \left(\frac{3}{7}+\frac{11}{14}\right) = (2+1)+\left(\frac{6}{14}+\frac{11}{14}\right)$

$= 3+\frac{17}{14} = 3+1\frac{3}{14} = 4\frac{3}{14}$

(b) $2\frac{1}{2} + 3\frac{2}{3}$

$$\begin{array}{r} 2\frac{1}{2} \\ +\,3\frac{2}{3} \\ \hline \end{array} \longrightarrow \begin{array}{r} 2\frac{3}{6} \\ +\,3\frac{4}{6} \\ \hline 5+\frac{7}{6} = \boxed{6\frac{1}{6}} \end{array}$$

Which of our number properties are represented in the rational numbers over addition?

Closure: $\frac{a}{b}+\frac{c}{d} \in \mathbb{Q}$

Commutative: $\frac{a}{b}+\frac{c}{d} = \frac{c}{d}+\frac{a}{b}$

Associative: $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f} = \frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$

Identity: $\frac{a}{b}+0 = 0+\frac{a}{b} = \frac{a}{b}$

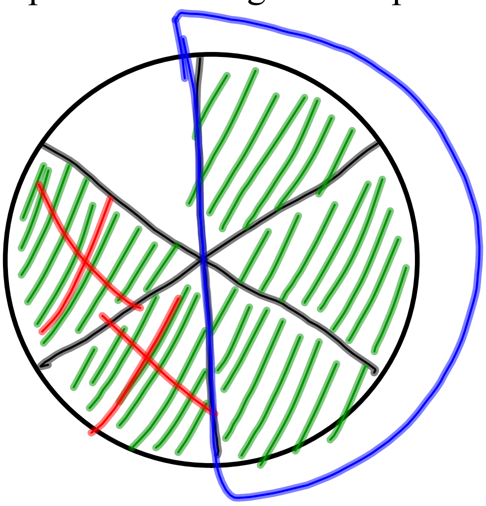
Inverse: $\frac{a}{b}+\frac{-a}{b} = \frac{-a}{b}+\frac{a}{b} = 0$

6.2 Completed Notes

Definition: If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then $\frac{a}{b} - \frac{c}{d}$ is the unique rational number $\frac{e}{f}$ such that $\frac{a}{b} = \frac{c}{d} + \frac{e}{f}$.

Note: An easy formula that we can get from this is $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$.

Example: Draw a figure to represent $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$.



Theorem: If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$.

(Note: In practice, we usually find the least common denominator instead of just using this formula.)

Proof: $\frac{a}{b} = \frac{a \cdot d}{b \cdot d} = \frac{ad}{bd}, \quad \frac{c}{d} = \frac{c \cdot b}{d \cdot b} = \frac{bc}{bd}$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad}{bd} - \frac{bc}{bd} = \frac{ad-bc}{bd}$$

Example: Calculate the following. Simplify your answer.

(a) $\frac{3}{5} - \frac{1}{4}$ $= \dfrac{12 - 5}{20} = \boxed{\dfrac{7}{20}}$

(b) $\frac{13}{16} - \frac{7}{30}$ $= \dfrac{13 \cdot 15}{16 \cdot 15} - \dfrac{7 \cdot 8}{30 \cdot 8} = \dfrac{195}{240} - \dfrac{56}{240} = \dfrac{139}{240}$

LCM is 240

Example: Calculate the following. Simplify your answer.

(a) $2\frac{1}{6} - 1\frac{9}{20}$

$$2\frac{1}{6} \to \cancel{2}^{1}\,\frac{\cancel{10}^{70}}{60}$$

$$-1\frac{9}{20} \to -1\frac{27}{60}$$

$$\boxed{\frac{43}{60}}$$

(b) $3\frac{1}{2} - 1\frac{5}{8}$

$$3\frac{1}{2} \to \cancel{3}^{2}\,\frac{\cancel{4}^{12}}{8}$$

$$-1\frac{5}{8} \to -1\frac{5}{8}$$

$$\boxed{1\frac{7}{8}}$$

6.2 Completed Notes

$$4\frac{2}{7} - 1\frac{5}{14}$$

$$\begin{array}{r} \overset{3}{\cancel{4}}\,\frac{\overset{18}{\cancel{4}}}{14} \\ -1\,\frac{5}{14} \\ \hline \boxed{2\frac{13}{14}} \end{array}$$